Efficient and accurate formulation of FE-based contact mechanics problems

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Abstract

Recent trend in the field of mechanical engineering designs promote the use of flexible multibody dynamics simulations as a crucial tool, in order to perform the required design analysis, especially when dealing with contact problems. Although parametric model order reduction techniques demonstrated great potential to deal with contact simulations, a true *hyper-reduction* of the multi-body contact is still not feasible. Moreover, non-smooth discretization used into the canonical Finite Element method leads to convergence issues in the analysis of sliding contact problems (e.g. gears and bearing) which is commonly compensated by usage of highly detailed meshes, whose level of detail is pushed far beyond the convergence requirements. In order to solve this fundamental conflict, this paper proposes to locally enrich the description of the contact boundaries, by means of the Hermite interpolants, in order to evaluate the contact forces on a finer detail. Preliminary results shows that the proposed approach is able to reduce the impact of the above mentioned conflict, allowing to reach the same quality of results with a coarser mesh, and a reduced computational impact.

1 Introduction

During the last decades, changes in environmental regulations faced mechanical engineers with a fundamental challenge: reducing pollution impact, without compromising performance. As a consequence, reduction of weight became one of the main design improvement drivers, due to its twofold advantages: reducing demand of material during manufacturing and improving efficiency during operational conditions.

In this evolving scenario, the use of flexible multibody dynamics simulations is becoming crucial to perform the required design analysis, especially when dealing with contact problems. In this problem class, single component deformations often result in significant variations of the contact topology (location and size of the contact area). Among others, parametric model order reduction (PMOR) techniques demonstrated great potential to deal with contact simulations. In [1], an efficient reduced-order model of a pair of contact gears was constructed by interpolating among a set of pre-computed contact shapes based on the angular configuration parameter of the gear pair, whereas in [2] the Static Modes Sliding approach [3] was extended to efficiently solve dynamic bearing contact problems.

In both above mentioned cases [1, 2], and more in general for (non-linear) Finite Element (FE) solution of contact problems [4], one fundamental challenge needs to be solved. Starting from a typical 3D solid mesh representation (e.g. tetrahedron, hexahedrons), the interacting contact surfaces are discretized using Lagrange elements (linear or quadratic surface shell) exhibiting C^0 continuity at the elements boundaries. In an isoparametric context this applies to both the geometry and the unknown displacement field. The non-smooth discretization leads to convergence issues in the analysis of sliding contact problems (e.g. gears and bearing) and to potentially highly oscillatory contact interactions even when mesh convergence is achieved. As a consequence, current solutions [1, 2] require to adopt highly detailed meshes. Besides, PMOR offers an efficient scheme to reduce problem dimensionality related to elastic deformation of the single components, but a true *hyper-reduction* of the multi-body contact is still not feasible, meaning that for some step of the solving process (especially the ones related to contact) the computational complexity still scales with the unreduced size of the problem. Summarizing, solution of contact problems is dominated by one fundamental conflict: on the one hand the level of detail of the meshes is pushed far beyond the convergence requirements, in order to accommodate the discontinuities arising when the contact zone cross the border of the existing FE; on the other hand, computational complexity of the existing methodologies demand for limiting the size of the mesh.

Similarly to existing surface smoothing techniques [5], Sauer and Corbett [6, 7] demonstrated the effectiveness of using higher order geometric description as an attractive alternative with respect to the otherwise highly detailed mesh required to maintain a good stability of the solution. In particular, they proposed a set of methodologies all aiming at locally enriching the existing finite elements, by creating ad-hoc element formulations for both 2D and 3D cases. Finally, they also assessed the effectiveness of the Hermite interpolation, with respect to Non-Rational Bi-Splines, emphasizing that their Hermite based element is able to offer the same performance. However, their contribution limited the use of Hermite only to 2D cases. This paper advances the state-of-the art by proposing an effective methodology to compute the Hermite interpolant for tridimensional surfaces.

In the scope of FE-based frictionless contact problems, this paper proposes an original approach for integrating the benefits of the Hermite interpolation rules within a canonical node-to-surf contact evaluation.

It relies on a local refinement of the existing mesh, aiming at locally increasing the level of discretization of the contact boundaries. The obtained more detailed mesh is then used within the canonical master/slave approach for the contact force evaluation, and the resulting detailed force distributions are later integrated on the original coarser mesh, by means of the bilinear shape functions.

The remainder of the paper is structured according to the following: Section 2 briefly introduces the context of frictionless contact mechanics, and in particular of the penalty formulations for contact problems. Section 3 describe the methodology to obtain the Hermite interpolants for a given bilinear mesh, whereas Section 4 focus on the integration of the Hermite patch into the node-to-surf contact algorithm. The impact of the proposed methodologies is discussed in Section 5, by means of a case study. Section 6 concludes the paper.

2 Frictionless contact problem within the Finite Element method

The solution of contact problems by means of the FE method involves several challenges, mostly due to the combination of the discontinuous nature of contact, with the discretized nature of the FE method. These problems tend to became even more challenging when the contact between two or more deformable bodies is under investigation.

The discontinuity of the contact phenomena among two generic bodies, b_i and b_j, is represented by the well-known Hertz-Signorini-Moreau condition [8]:

$$\mathcal{G}_{ij} \geq 0, \quad \mathcal{P}_{ij} \leq 0, \quad \mathcal{P}_{ij}\, \mathcal{G}_{ij} = 0 \tag{1}$$

Where the contact force acting on the two bodies surfaces, $\mathcal{P}_{ij}$, is *activated* whenever the two bodies get into contact (i.e. the gap, $\mathcal{G}_{ij}$, becomes null), whereas it becomes *inactive* ($\mathcal{P}_{ij} = 0$), as long as the minimum directed distance between the bodies is positive.

A large variety of formulations has been presented so far in order to deal with contact problems [8]. Within the broader context of flexible multibody systems, effective manipulation of the contact interaction demands for the concurrent solution of two main challenges. First, a collision detection strategy needs to be adopted in order to continuously distinguish between the two above mentioned states of equation (1), and this for each of the possible contact pairs within the system. Second, the adopted formulation needs to accommodate for the discontinuous nature of the contact constraints.

This paper follow the penalty method [8], whose main idea is to introduce into the equation of motion of the system a fictitious stiffness term whose goal is to mimic the effect of the non-penetrability conditions, as resulting from the activation of (1). This is achieved by implementing a master/slave node to surface [9], whose detail are given into Section 2.1.

2.1 The node-to-surface penalty approach

Given the full deformation state for two bodies, the evaluation of the contact forces necessary to comply with the Signorini condition (1), is accomplished by the execution of the following steps. For each node Q belonging to the slave body surface, the penetration gap is computed by finding the point on the surface of the i^{th}-master element having the smallest Euclidean distance to the slave node Q (see Fig. 2).

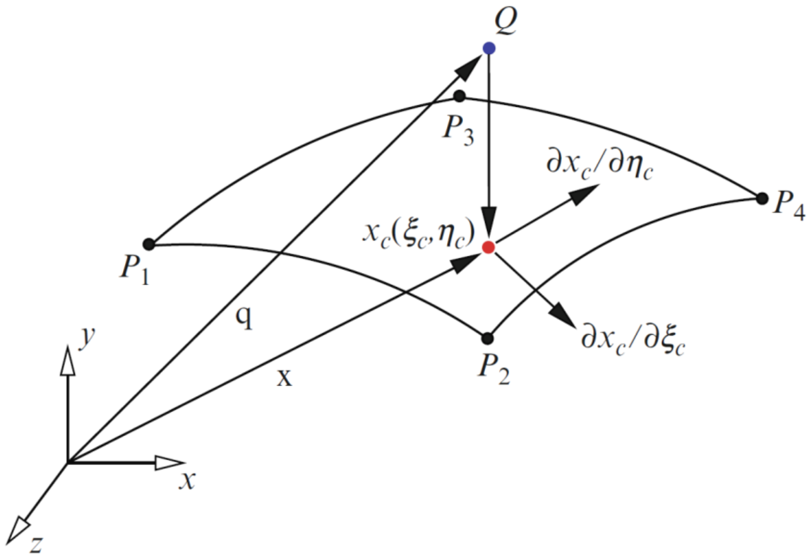


Figure 1: Node to surface minimal distance problem for the isoparametric bilinear element

In general, the above stated problem reduces to the solution of a system of equation obtained by imposing the orthogonality condition between the vector $\overrightarrow{Q - x_c}$ and the parametric tangents to the surface $\mathcal{S}$ passing through x_c along the direction of the variation of each isoparametric coordinate p_i:

$$\frac{\partial \mathcal{S}}{\partial p_i} \cdot \overrightarrow{Q - x_c} = 0 \tag{2}$$

The resulting system of equations, can be solved, by means of an iterative Newton-Raphson scheme.

Once the parameters p_i are calculated, the minimal distance g_N between the two bodies is found by taking the projection of $\overrightarrow{Q - x_c}$ along the normal pointing inwards the master element, n_c:

$$g_N = n_c \cdot (x_C - Q) \tag{3}$$

Finally, the penalty contact force acting on the slave node can be computed as:

$$F_c = \begin{cases} c_p\, g_N\ n_c, & g_N < 0 \\ 0, & otherwise \end{cases} \tag{4}$$

Figure (2) exemplifies the above described problem for the case of a quadrangular bilinear surface in 3D, where the common isoparametric coordinates ξ and η are used instead of the abstract notation p_i.

3 The Hermite Bicubic Enrichment for a quadrangular patch

This section discusses the derivation of the Hermite interpolants, which are used for the local refinement of an existing FE mesh. To this aim, a preliminary step need to be executed during which the normal to the surface for each of the existing mesh nodes needs to be evaluated, even when considering a deformed configuration. This is achieved by considering the average of all the normal versors obtained for the same node as the cross products of each pair of edges concurring to it (e.g. for the node 3 of the quadrangular element depicted in Figure (2), first the normal N_3^i is obtained by the cross product of $\overrightarrow{S_{32}}$ versus $\overrightarrow{S_{34}}$; second, all the other normal, concurring to the same node are evaluated; finally the normal N3 is evaluated as the average value of the previously computed versors).

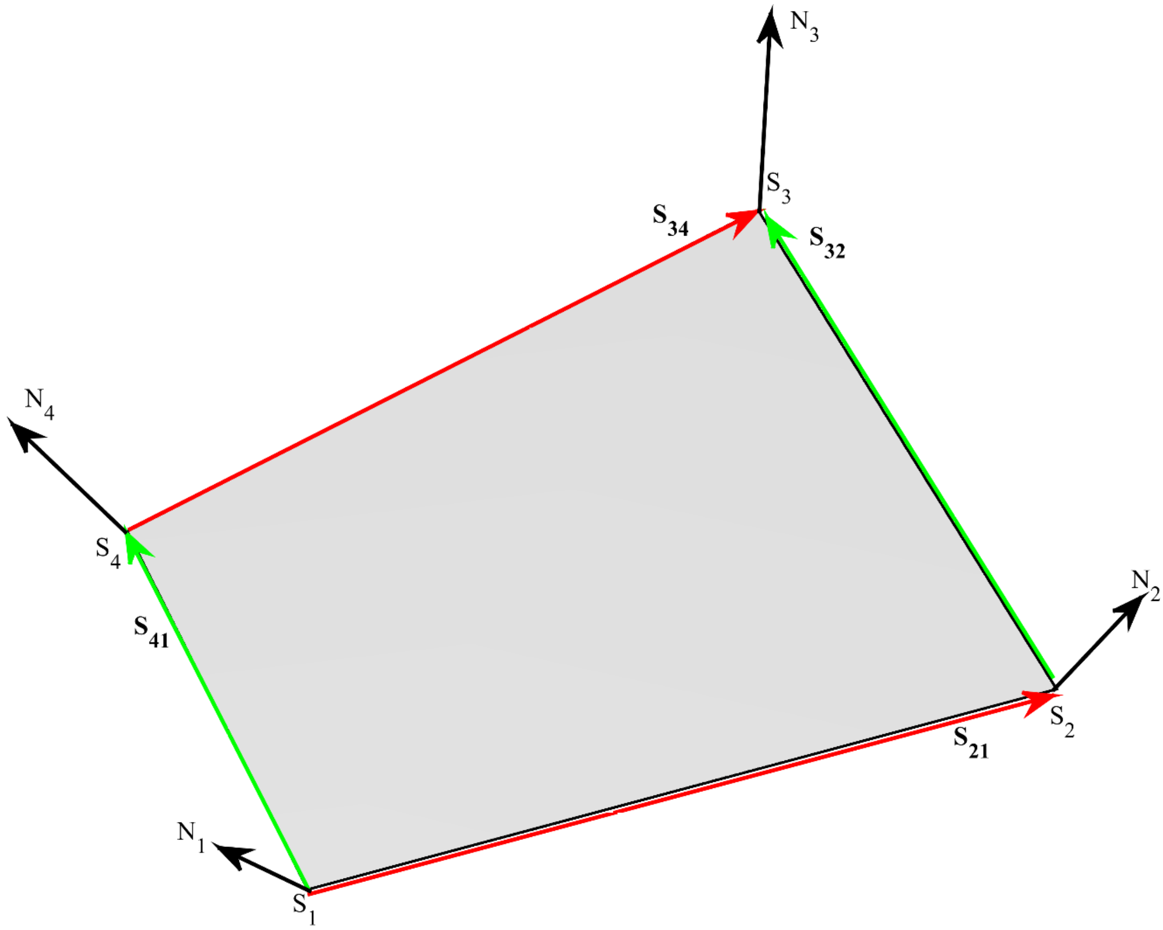


Figure 2: Bilinear surface element, used as the input for computing the Hermite patch

In this work, but without loss of generality, we limit ourselves to the derivation of the Hermite interpolant for an existing bilinear element. The problem can be stated as that of finding the coefficients of the bicubic interpolation polynomials defining each coordinate of the parametric bicubic surface $\mathcal{S}(u,v)$:

$$\mathcal{S}(u,v) \equiv \{\mathcal{S}^x(u,v), \mathcal{S}^y(u,v), \mathcal{S}^z(u,v)\} \tag{5}$$

Where

$$\mathcal{S}^k(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}^k\, u^i v^j\,, with{:}\ k = \{x,y,z\} \tag{6}$$

Equation (6) can be also rearranged in a more compact matricial form:

$$\mathcal{S}^k(u,v) = U\, A^k\, V' \tag{7}$$

where the isoparametric vectors, U and V, depend respectively on the two isoparametric coordinates, u and v:

$$U = [1\; u\; u^2\; u^3], \quad V = [1\; v\; v^2\; v^3] \quad with\, u, v \in \mathbb{R} \wedge 0 \le u, v \le 1 \tag{8}$$

As stated in (5) and (6), three different interpolation matrices, A^k, need to be found in order to fully define the parametric surface. As reported in [10], the derivation of the Hermite patch can be developed imposing C^0 and C^1 over the nodes and the edges of the isoparametric element. The solution to the problem is given as:

$$\mathcal{S}^k(u, v) = U\, A^k\, V' = U\, M\, B^k\, M'V' \tag{9}$$

where the interpolation matrices A^k are now expressed as a function of the Hermitian basis, M_h:

$$M_h = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \tag{10}$$

The matrices B_h^k are derived according to the imposed boundary conditions. In this work we rely on the Ferguson condition [11], which allows one to define the Hermite surface by only prescribing 12 boundary conditions, leading to the following form of the interpolation matrix, B_h:

$$B_h = \begin{bmatrix} S_1 & S_2 & S_1^v & S_2^v \\ S_4 & S_3 & S_4^v & S_3^v \\ S_1^u & S_2^u & 0 & 0 \\ S_4^u & S_3^u & 0 & 0 \end{bmatrix} \tag{11}$$

As a results, the bicubic patch expression already given in (9) can be expressed as a function of the position of the four corners, S_i, and their partial derivative with respect to the two isoparametric coordinates:

$$S_i^u = \frac{\partial \mathcal{S}}{\partial u}, \qquad S_i^v = \frac{\partial \mathcal{S}}{\partial v} \tag{12}$$

As depicted in Figure (3), S_i^u and S_i^vrepresent two tangent to the surface, $\mathcal{S}$, defined at each of the four corner nodes. As such, they are always coplanar with the normal to the surface, N_i , and the local variation of the corresponding isoparametric coordinates, S_{ij} , for which the following notation was used:

$$S_{ij} = S_i - S_j \tag{13}$$

whose derivation is also illustrated in Figure (2). This notion allows to derive them according to the following set of cross products:

$$S_1^u = N_1 \times\, S_{21} \times N_1 \tag{14}$$

$$S_2^u = N_2 \times\, S_{21} \times N_2 \tag{15}$$

$$S_3^u = N_3 \times\, S_{34} \times N_3 \tag{16}$$

$$S_4^u = N_4 \times\, S_{34} \times N_4 \tag{17}$$

$$S_1^v = N_1 \times\, S_{41} \times N_1 \tag{18}$$

$$S_2^v = N_2 \times\, S_{32} \times N_2 \tag{19}$$

$$S_3^v = N_3 \times\, S_{32} \times N_3 \tag{20}$$

$$S_4^v = N_4 \times\, S_{41} \times N_4 \tag{21}$$

Finally, the locally refined mesh is obtained evaluating the Hermite surface (1) with the desired Level of Detail Magnification (LoDM). Figure (3) illustrates the refined mesh obtained starting from the underlying

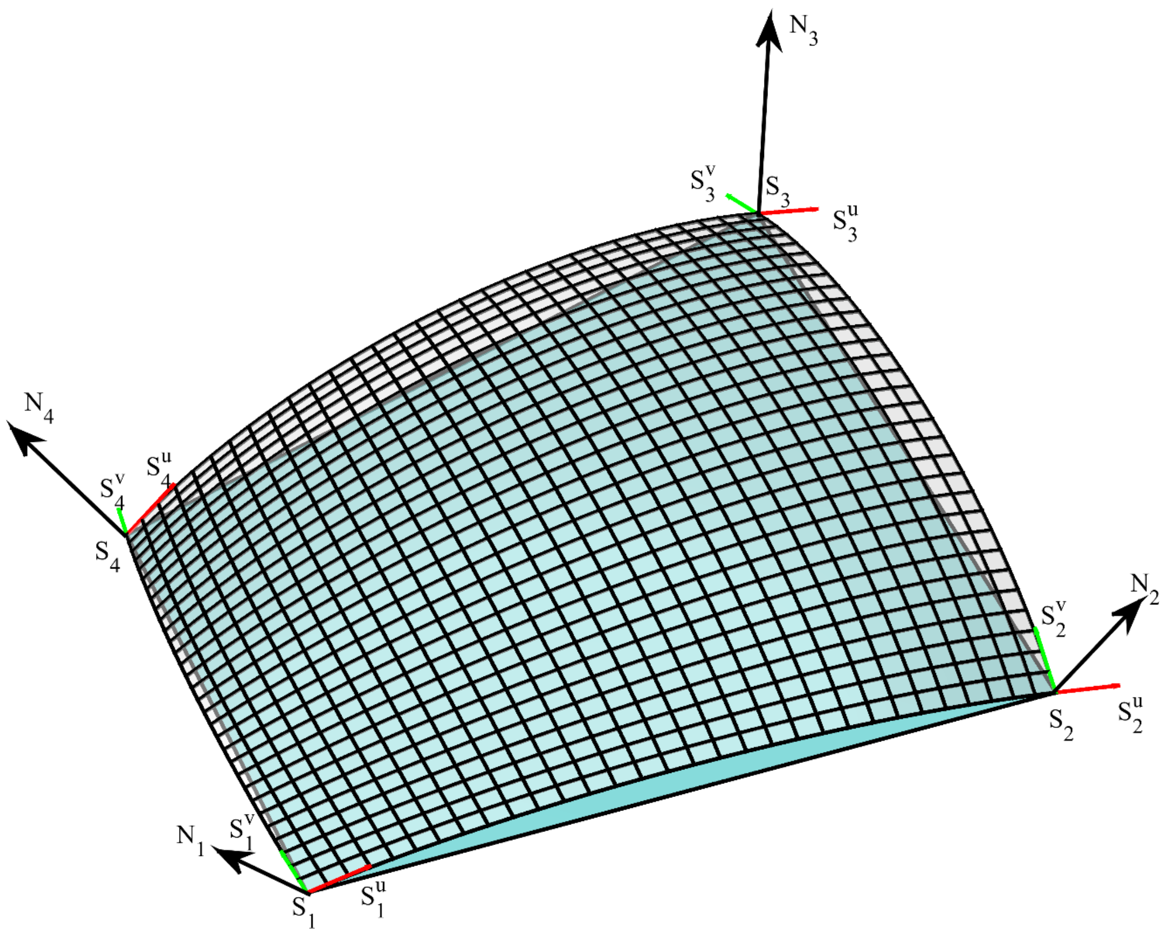


Figure 3: Bilinear locally refined mesh (1024x)

bilinear element, with a LoDM of 1024x (i.e. the original element was refined using a detailing grid of 32 by 32).

By exploiting the methodology explained so far, the contact force evaluations can be assessed by using a mesh with an arbitrary finer level of discretization. In order to use the resulting detailed set of contact forces, a bilinear interpolation can be adopted in order to transfer back the forces from the detailed mesh to the original one. In this way the weight given to the nodal forces for a given nodal penetration will be representative of a detailed contact geometry

4 Results

In order to assess the impact of the approach proposed in section 3, a set of numerical experiment has been carried out where the main goal was to determine the static equilibrium between two identical spur gears, under the effect of a given torque value applied to the driving gears, while maintaining fixed the rotational degree of freedom of the other one.

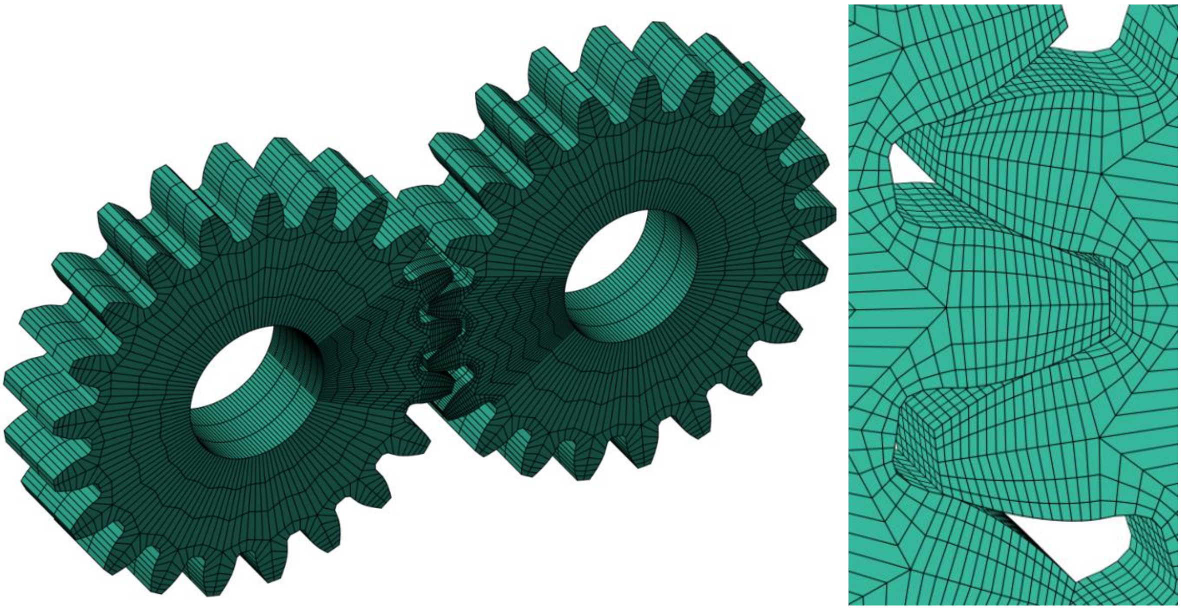
A set of four mesh has been generated for the gear pairs, where only the number of element over the flank has been varied, in order to build a controlled increase in the level of the detail. In particular, the four meshes used 8, 16, 32 and 64 elements over the teeth profile, and will be referred respectively as FE linear 8, FE linear 16, FE linear 32, and FE linear 64. The main geometrical parameters of the used gear are summarized in Table 1.

The solution of the contact problem involving a gear pair has been carried out by following the methodology proposed in [1], and implemented in the in-house code MUTANT – Multibody Transient Analysis of

Transmissions, available at the KUL PMA department [11]. All the simulation have been executed on a Dell Precision M6800 workstation, equipped by i7-4710MQ CPU, with 32 GB of RAM.

Number of teeth	21
Normal module	4
Normal pressure angle [deg]	20
Addendum modification factor	1
Dedendum modification factor	1.25
Density [$\boldsymbol{Kg/m^3}$]	7810
Young's modulus [GPa]	210
Poisson's ratio	0.3

Table 1: Gear characteristics



(a) (b)

Figure 4: Visual representation of the FE mesh of the gears (a) and a detailed view of the contact area (b)

For each of the above mentioned mesh, several simulation have been carried out with different values of LoDM. Table 2 summarizes the set of all comparative tests (namely E16, E32, and E64) that have been carried out. For each set of tests, the LoDM have been tuned such to reach the same level of detail during the contact evaluation stage.

Mesh\LoDM	1x	2x	4x	8x
FE linear 8	**E8**	**E16**	**E32**	**E64**
FE linear 16	**E16**	**E32**	**E64**	
FE linear 32	**E32**	**E64**		
FE linear 64	**E64**			

Table 2: Overview of the comparative tests

An indication of the computational complexity can be observed by looking at the processing time required for solving one of the equilibrium configuration (as indicated at the bottom chart in figure 3). It is worth noting that the computational complexity scale always more favorably for an increase of the LoDM. E.g. taking as a reference the mesh <FE linear 16> with LoDM <1x>: in order to double the global level of detail, it is more convenient to increase the LoDM to <2x>, than just producing a new mesh (namely the <FE linear 32>), although this produces the risk of using a mesh that is not representative of the true stiffness.

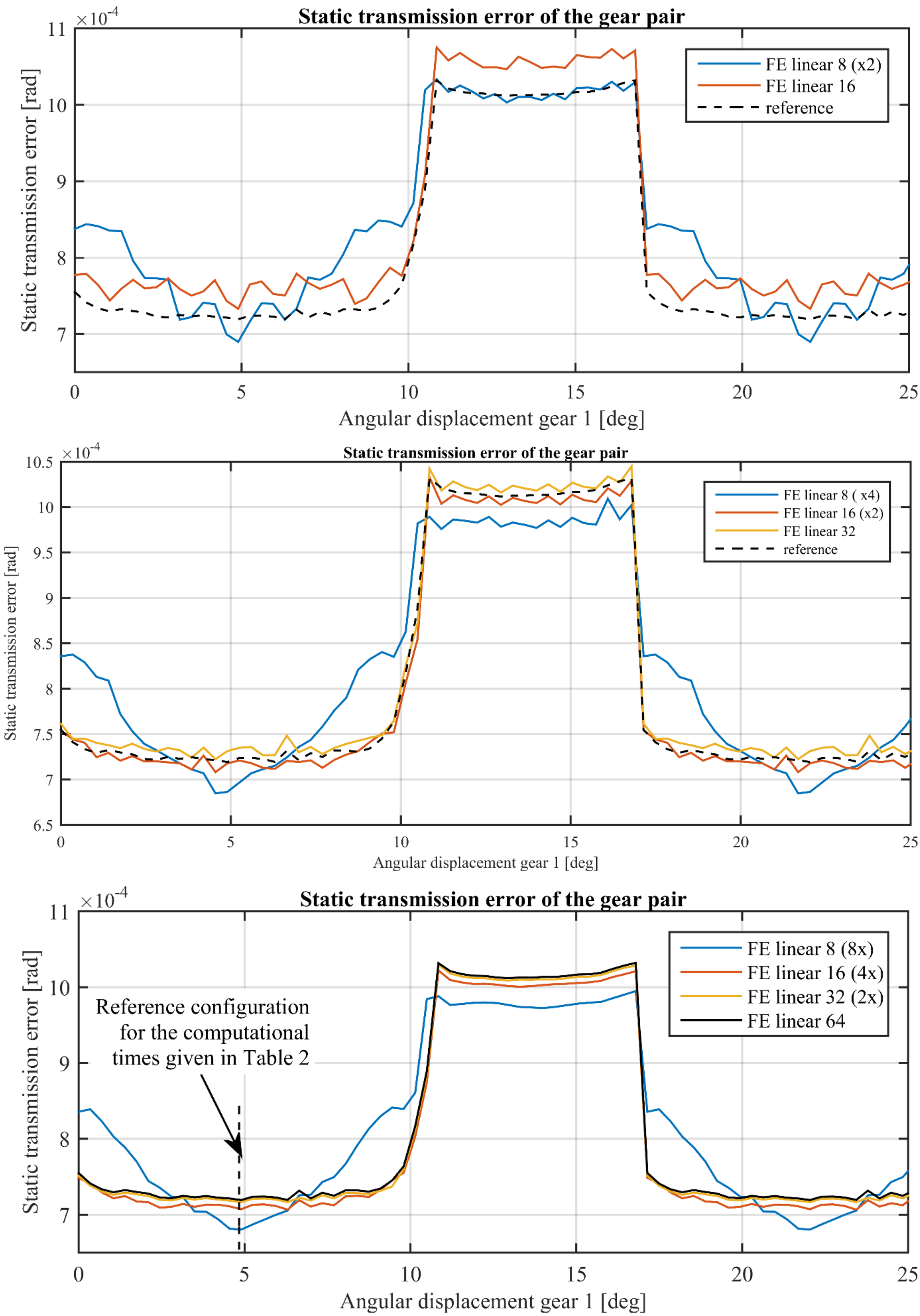


Figure 5: Static transmission error simulated for increasing levels of discretization over the gears flank: 16 div (top), 32 div (mid), 64 div (bottom).
In each legend the LoDM factor used is given in parenthesis

Mesh size	$\boldsymbol{ref}(\mathbf{1}\boldsymbol{x})$	$\mathbf{2}\boldsymbol{x}$	$\mathbf{4}\boldsymbol{x}$	$\mathbf{8}\boldsymbol{x}$
8	4.9	8.2	10.9	18.3
16	17.2	25.0	29.0	
32	37.0	45.6		
64	>100	-	-	-

Table 3: Processing time [secs] for computing the kinematic equilibrium with two teeth in contact, as indicated in figure (2), for different level of mesh details and different level of refinement

In figure (5), a comparison of the STE obtained for the three different level of desired details is depicted.

From all the conditions, it is evident that the coarser mesh <FE Linear 8> is not detailed enough to provide the proper solution. However, especially for the coarser meshes, the proposed methodology is able to smooth out the undesired unphysical high-frequency component (in spatial domain) in the resulting deformation. Finally, it is worth noting that the mesh labeled as <FE Linear 16> is able to reach the same quality of the finest mesh, when combined with the proper LoDM. This results, combined with the computational performance discussed in Table 3 allows to conclude that the use of the proposed surface enrichment can lead to a big improvement of the overall quality of the results, without sacrificing computational performances. This is achieved by alleviating the undesired oscillatory behavior of the penalty integration scheme combined with the discontinuity of the used discretization.

5 Conclusions

Solution of contact problems is dominated by one fundamental conflict: on the one hand the level of detail of the meshes is pushed far beyond the convergence requirements, in order to accommodate the discontinuities arising when the contact zone cross the border of the existing FE; on the other hand, computational complexity of the existing methodologies demand for limiting the size of the mesh. Aiming at mitigating the effects of this conflict, this paper proposed an original methodology to locally enrich the degree of description of the contact boundary. Local refinement of the contact surfaces is realized by means of Hermite interpolants.

Results of the proposed approach confirmed the great potential of the proposed methodology by showing a reduction of the overall computational complexity, and unexpectedly suggest the feasibility of using a reduced level of detail for the FE mesh involved in contact problems.

Further research will investigate the possibility to involve the Hermite surface description directly into the contact evaluation, such to exploit the continuity of the enriched boundary description thus eliminating the computational challenges posed from current discretization of the contact boundaries.

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